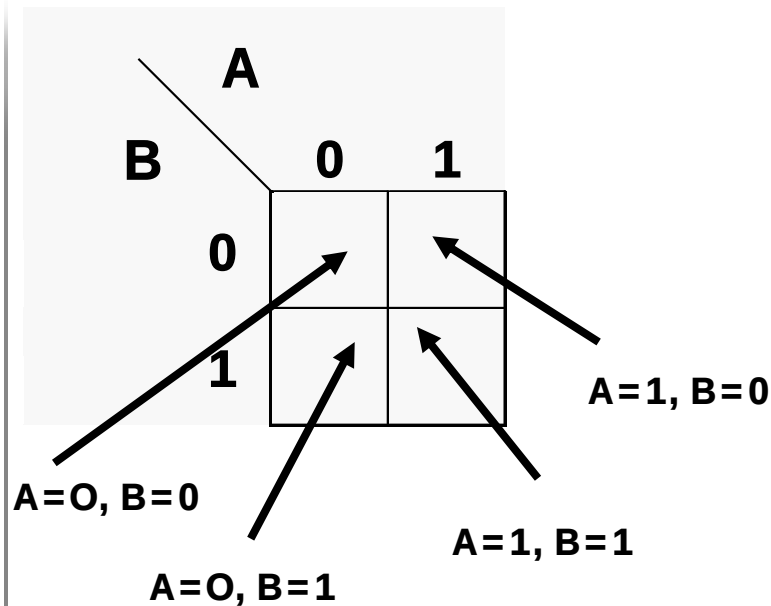


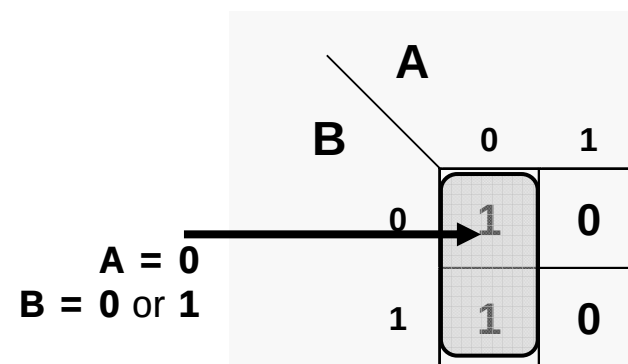
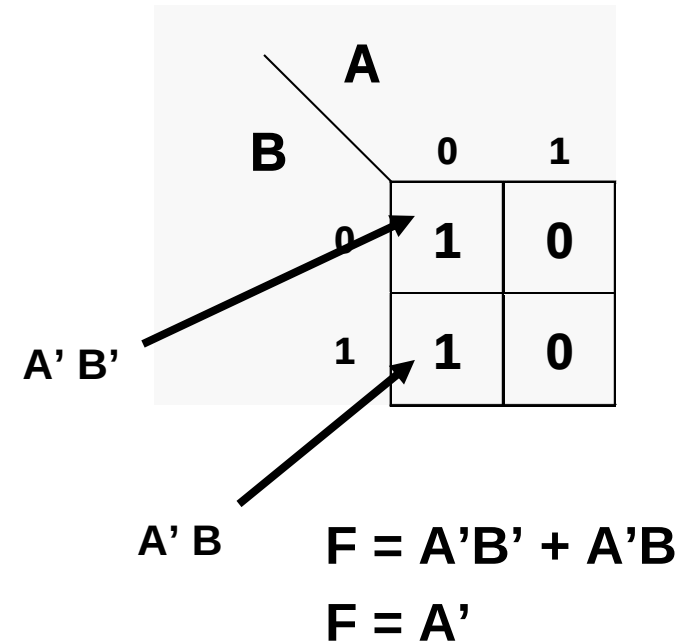
What are Karnaugh Maps?

- A simpler way to handle most (but not all) jobs of manipulating logic functions.
- The map is a diagram made up of squares, each small square represents a minterm. Any function is made up of minterms and can be represented on the map. Adjacent minterms can be combined to form simpler terms.
- Karnaugh Map Advantages
 - Can be done more systematically
 - Much simpler to find minimum solutions
 - Easier to see what is happening

2 Variable Karnaugh Map



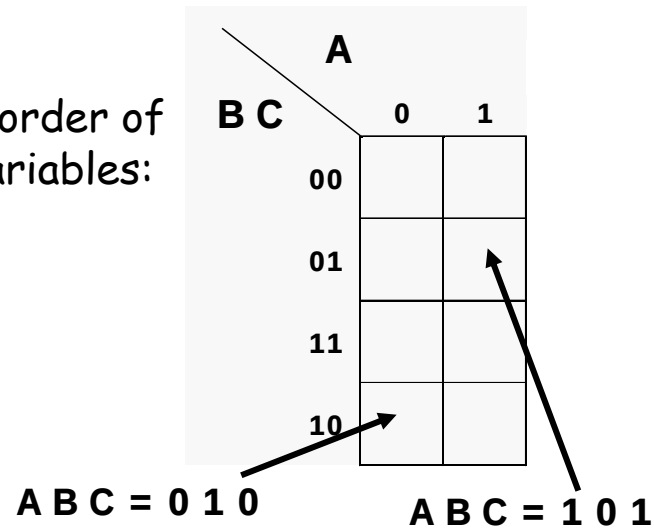
A	B	F
0	0	1
0	1	1
1	0	0
1	1	0



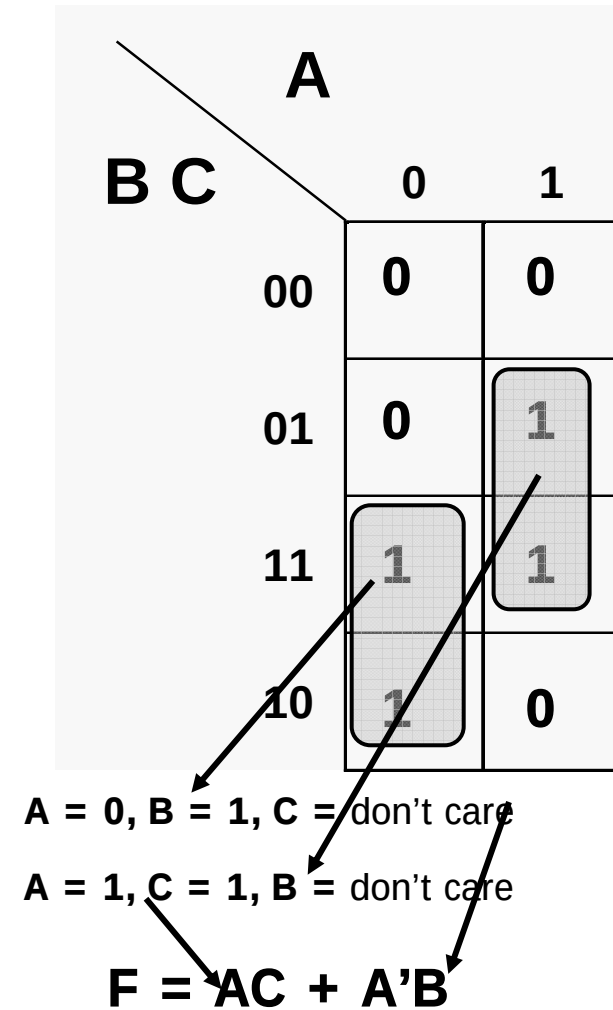
3 Variable Karnaugh Map

Note the order of the B C variables:

0 0
0 1
1 1
1 0



A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1



Minterm Expression to Karnaugh Map

$$F(A,B,C) = \sum m(1, 4, 5, 6)$$

		A	
		0	1
B C	00	0	1
	01	1	1
	11	0	0
	10	0	1

Arrows from the minterm expression point to the 1s in the Karnaugh Map:

- 1 (minterm 1) points to cell (01, 0)
- 4 (minterm 4) points to cell (10, 0)
- 5 (minterm 5) points to cell (01, 1)
- 6 (minterm 6) points to cell (10, 1)

Remaining map spaces are 0.

		A	
		0	1
B C	00	0	4
	01	1	5
	11	3	7
	10	2	6

Each minterm corresponds to a location on the Karnaugh Map

$m_0 \dots m_7$

Maxterm Expression to Karnaugh Map

Each maxterm in the expression corresponds to a 0 on the Karnaugh Map

$M_0 \dots M_7$

		A	
		0	1
B C	00		
	01		
	11		
	10		
		2	6

$$F(A,B,C) = \prod M(2, 3, 4)$$

		A	
		0	1
B C	00	1 <i>0</i>	0 <i>4</i>
	01	1 <i>1</i>	1 <i>5</i>
	11	0 <i>3</i>	1 <i>7</i>
	10	0 <i>2</i>	1 <i>6</i>

Remaining map spaces are 1

Adjacent Spaces

		A	
		0	1
B C	00		$AB'C'$
	01	$A'B'C$	$AB'C$
	11		ABC
	10		

Adjacent spaces on the map differ by complementing exactly one variable.

		A	
		0	1
B C	00		$AB'C'$
	01		
	11		
	10		ABC'

Note that the map “wraps around” so edge spaces are also adjacent.

Combining Terms

Adjacent spaces can be combined using: $XY + XY' = X$

		A	
		0	1
B C	00		AB'C'
	01	A'B'C	AB'C
	11		
	10		ABC'

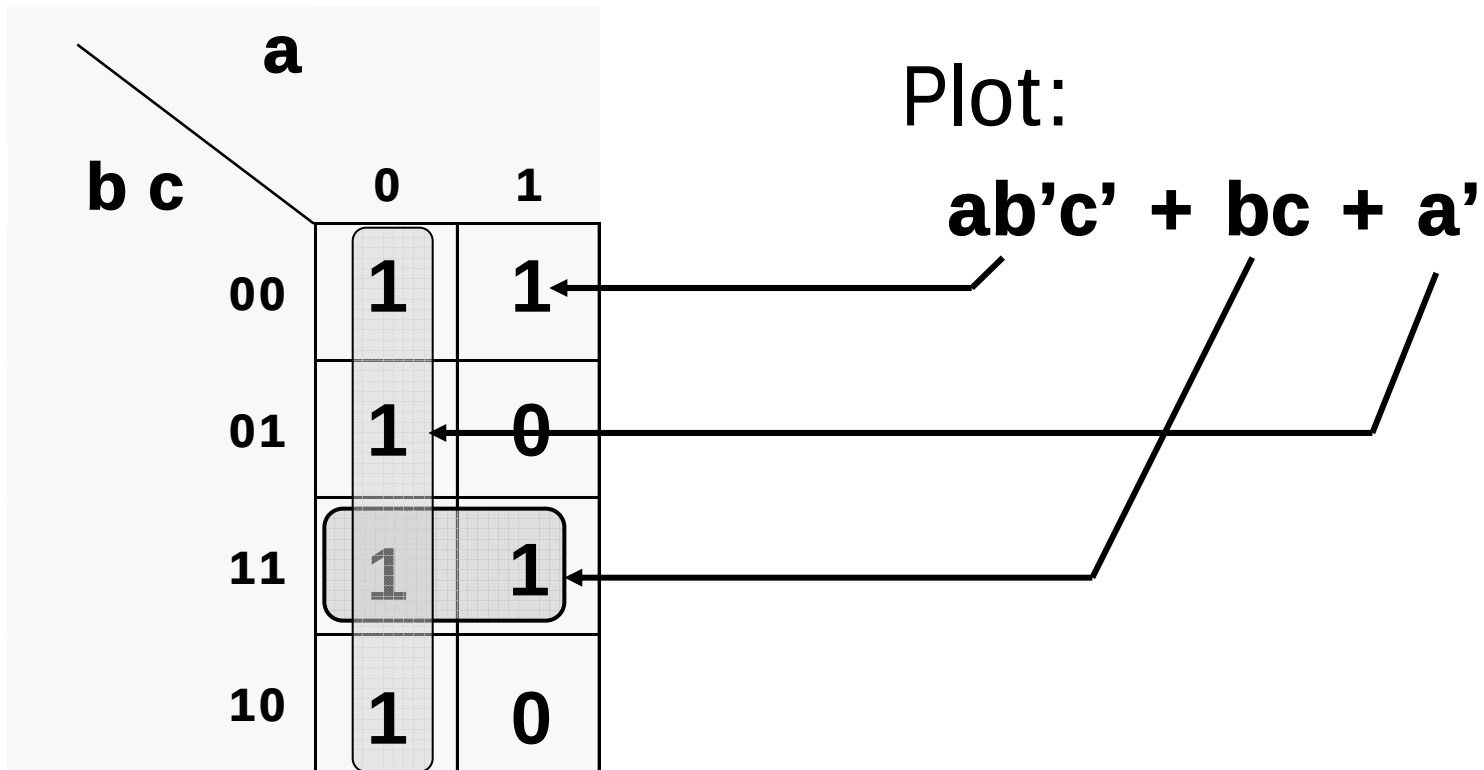
$$A'B'C + AB'C = B'C$$

$$AB'C' + ABC' = AC'$$

		A	
		0	1
B C	00	A'B'C'	AB'C'
	01	A'B'C	AB'C
	11		
	10		

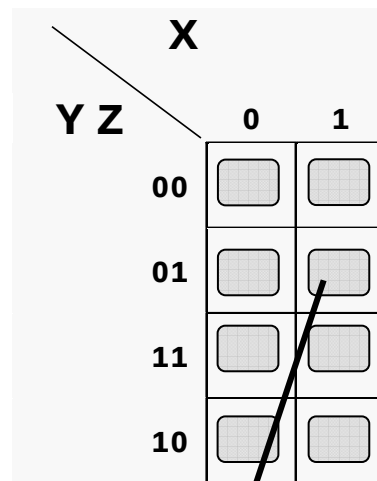
$$\begin{aligned}
 &A'B'C' + AB'C' + A'B'C + AB'C \\
 &B'C'(A + A') + B'C(A + A') \\
 &B'(C + C') \\
 &B'
 \end{aligned}$$

Boolean Algebra to Karnaugh Map



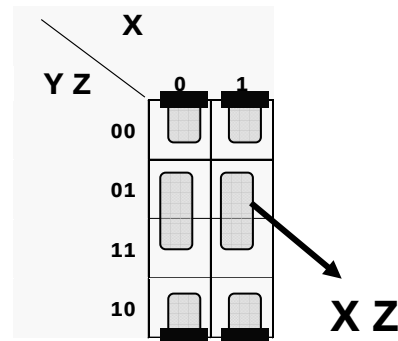
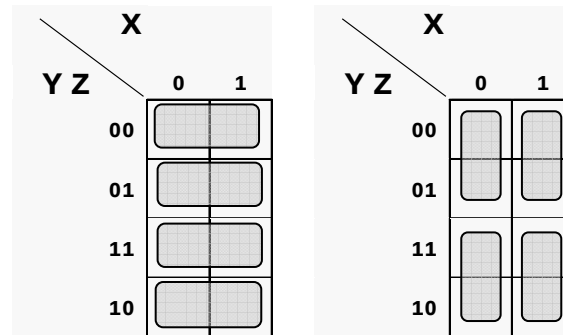
Mapping Sum of Product Terms

The 3 variable map has 8 possible terms with 3 literals

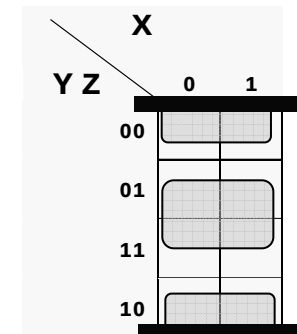
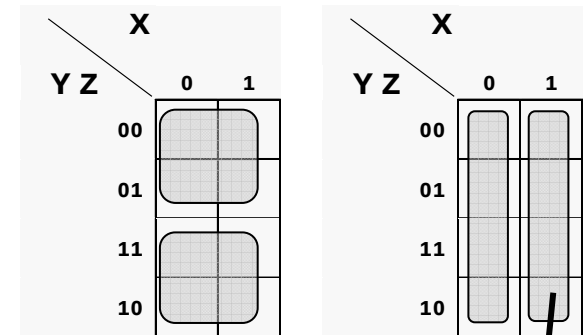


XY'Z etc.

The 3 variable map has 12 possible terms with 2 literals



The 3 variable map has 6 possible terms with 1 literal



X